Typo:

$$r_p = 2R \longrightarrow \vec{r}_p = 2R\hat{x}$$

should be

$$V = -\int_a^b \vec{E} \cdot d\vec{l}$$

Show that over a closed circuit; the potential is zero.

$$V = -\oint_c \vec{E} \cdot d\vec{l} = 0$$

Proof:

$$\oint_c \vec{E} \cdot d\vec{l} = \oint_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = 0$$

* Show that $\vec{\nabla} \times \vec{E} = 0$ everywhere in electrostatic.

$$\oint_c \vec{E} \cdot d\vec{l} = 0$$

$$\longrightarrow \int_a^b E \cdot d\vec{l}$$ does not depend on path.

Point charge Q at $\vec{r}_p$

$$\vec{E}(r_p) = \frac{Q}{4\pi\varepsilon_0} \frac{(\vec{r}_p - \vec{r}_s)}{|\vec{r}_p - \vec{r}_s|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{(x_p - x_s)\hat{x} + (y_p - y_s)\hat{y} + (z_p - z_s)\hat{z}}{[(x_p - x_s)^2 + (y_p - y_s)^2 + (z_p - z_s)^2]^{3/2}}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x_p} & \frac{\partial}{\partial y_p} & \frac{\partial}{\partial z_p} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\hat{x}\left(\frac{\partial}{\partial y_p} E_y - \frac{\partial}{\partial z_p} E_z\right) + \hat{y}\left(\quad \right) + \hat{z}\left(\quad \right)$$

$$\frac{\partial}{\partial z_p} E_z = \frac{\partial}{\partial y}\left[\frac{Q}{4\pi\varepsilon_0} \frac{(z_p - z_s)}{[(x_p - x_s)^2 + (y_p - y_s)^2 + (z_p - z_s)^2]}\right]$$

$$= \frac{2Q}{4\pi\varepsilon_0}(y_p - y_s)(z_p - z_s) - \frac{3}{2}\left[\frac{1}{(\qquad)^{5/2}}\right]$$

$$\frac{\partial}{\partial z_p} E_y = \frac{\partial}{\partial z_p}\left[\frac{Q}{4\pi\varepsilon_0}\left(\frac{y_s - y_p}{[(x_p - x_s)^2 + (y_p - y_s)^2 + (z_p - z_s)^2]^{3/2}}\right.\right.$$

$$= \frac{\partial}{\partial z_p} E_z$$

here we see that for all curl of the electrostatic feild is zero.

$\therefore$ For electrostatic Feilds

$$\vec{\nabla} \times \vec{E} = \emptyset$$

* $$\oint_c \vec{E} \cdot d\vec{l} = \oint (\vec{\nabla} \times \vec{E}) \cdot d\vec{S} = \emptyset$$

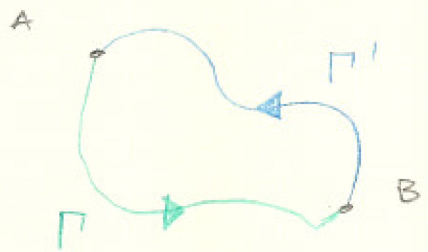
$\therefore$ For Static feilds

going from $a \to a$ there is no work done.

QED

EX:

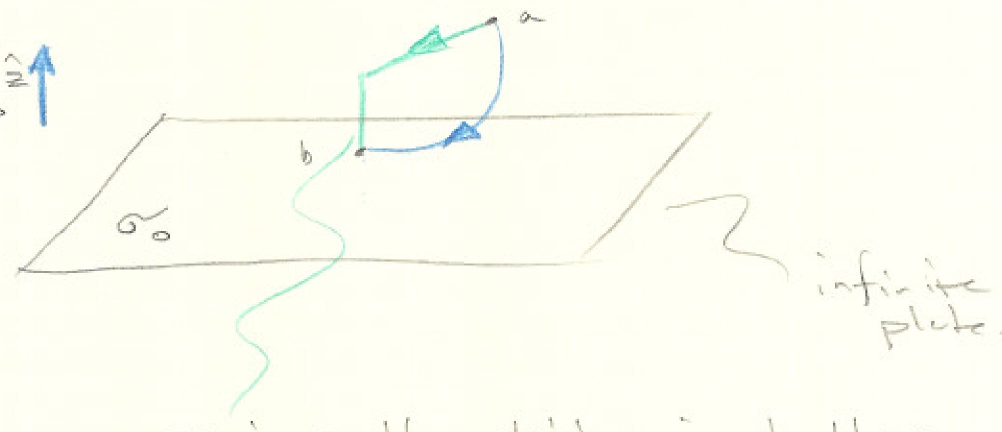
$$V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l}$$



note: we know that

$V_{ab} - V_{ba} = 0$

it does not depend on Γ'



$\frac{\sigma_0}{2\varepsilon_0}\hat{z}$

This path choice is better since going perpendicular to the plate the cross products are zero remember that this is not path dependant.

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \times \overrightarrow{(\text{something})} = 0$$

$$\therefore \overrightarrow{(\text{something})} = \vec{\nabla} \times \overrightarrow{(\text{something else})}$$

$$\therefore \vec{\nabla} \times \vec{\nabla} \times \overrightarrow{(\text{something})} = 0$$

Special Case

$$\vec{E} = E_x \hat{x}$$

$$V = -\int \vec{E} \cdot d\vec{l} \Leftrightarrow dV = -E_x dx$$

$$E_x = -\frac{dV}{dx}$$

$$= -\vec{\nabla} V \Big|_x$$

Something else $= -V$

$$\vec{E} = -\vec{\nabla} V$$

Properties of $\vec{\nabla}$ (geometric)

* $\vec{\nabla} V$ is perpendicular to lines of constant V
* Points in the direction of greatest i/c in V